Solutions of Yamabe Equation via Method of Moving Spheres

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From 3.00 p.m. to 3.45 p.m
Room: *** To Be Confirmed ***

Abstract

The primary objective of this seminar activity is to present a rigorous proof characterizing the solutions which are non-negative and entire (i.e. "defined on the whole space $\mathbb{R}^n$", with $n \geq 3$) of the Semi-Linear Elliptic Partial Differential Equation, commonly referred to as the "Yamabe Equation" and represented as follows:

$$\Delta u = -n(n-2)u^{\frac{n+2}{n-2}}. \tag{YE}$$

In particular, after a concise explanation of the analytic origins underlying the significance of (YE), and after few considerations about some evident symmetries that non-negative entire solutions must satisfy, we will state and motivate a very deep theorem which implies some remarkably strong properties of the solutions we are considering.

In conclusion, leveraging the aforementioned theorem, recognized as the "Moving Sphere Method", the seminar will provide a condensed exposition of the proof establishing that the non-negative entire solutions of the Yamabe Equation are exclusively those functions belonging to the set defined as:

$$\left\{ u : \mathbb{R}^n \to \mathbb{R}^+ \,\middle|\, u(x) = \left(\frac{\lambda}{\lambda^2 + |x - x_0|^2} \right)^{\frac{n-2}{2}} , \ \lambda > 0, \ x_0 \in \mathbb{R}^n \right\}.$$

Bibliography

[1] M. Fraccarolli, *L'equazione di Yamabe: analisi e soluzioni attraverso il Metodo delle Sfere Mobili e il Principio del Massimo*, Univeristy of Padua, 2014

[2] Y. Li and L. Zhang, *Liouville type theorems and Harnack type inequalities for semilinear elliptic equations*, J. Anal. Math. 90 , 27–87, 2003.